1 INTRODUCTION

Cams are used for essentially the same purpose as linkages, that is, generation of irregular motion. Cams have an advantage over linkages because cams can be designed for much tighter motion specifications. In fact, in principle, any desired motion program can be exactly reproduced by a cam. Cam design is also, at least in principle, simpler than linkage design, although, in practice, it can be very laborious. Automation of cam design using interactive computing has not, at present, reached the same level of sophistication as that of linkage design.

The disadvantages of cams are manufacturing expense, poor wear resistance, and relatively poor high-speed capability. Although numerical control (NC) machining does cut the cost of cam manufacture in small lots, costs are still quite high in comparison with linkages. In large lots, molding or casting techniques cut cam costs, but not to the extent that stamping and so forth, can cut linkage costs for similar lot sizes. Unless roller followers are used, cams wear quickly. However, roller followers are bulky and require larger cams, creating size and dynamic problems. In addition, the bearings in roller followers create their own reliability problems. The worst problems with cams are, however, noise and follower bounce at high speeds. As a result, there is a preoccupation with dynamic optimization in cam design. Cam design usually requires two steps (from a geometric point of view):

1. synthesis of the motion program for the follower and
2. generation of the cam profile.

If the motion program is fully specified throughout the motion cycle, as is the case, for example, with the stitch pattern cams in sewing machines, the first step is not needed. More usually, the motion program is specified only for portions of the cycle, allowing the synthesis of the remaining portions for optimal dynamic performance. An example is the cam controlling the valve opening in an automotive engine. Here the specification is that the valve should be fully closed for a specified interval and more or less fully open for another specified interval. For the portions of the cycle between those specified, a suitable program must be synthesized. This can be done, with varying levels of sophistication, to make the operation of the cam as smooth as possible. In general, the higher the level of dynamic performance required, the more difficult the synthesis process.

The second stage of the process, profile generation, is achieved by kinematic inversion. The cam is taken as the fixed link and a number of positions of the follower relative to the cam is constructed. A curve tangent to the various follower positions is drawn and becomes the cam profile. If the process is performed analytically, any level of accuracy can be achieved.

8.2 CAM-FOLLOWER SYSTEMS

A general cam-follower system consists of three elements as shown in Fig. 8.1. The first two are the cam and follower, and the third is a spring or other means of ensuring that the follower remains in contact with the cam. The function of the spring can be replaced by gravity or by constraining the follower between the two surfaces on the cam or constraining the cam between two surfaces on the follower. Both of these approaches are usually more expensive than using a spring and therefore are not commonly used. A follower is characterized by its motion relative to the ground link and by the geometry of its face that
between roller and rigid cylindrical-faced followers. Obviously, there is a significant difference from an overall design standpoint, however. Although here we will consider only planar, rotating cams, in practice a large number of different cam geometries are found. Some of the different types of cams and follower systems are shown in Fig. 8.4.
contacts the cam. The cam-follower motion may be either rotational or translational, and translating followers may be either radial or offset. Examples of these are shown in Fig. 8.2. The follower surfaces may be either knife edged, flat, spherical (or cylindrical), or roller as shown in Fig. 8.3. Actually, these geometries are all of the same class depending on the radius of curvature of the follower face. That is, the knife edge has a radius of curvature that is zero, the flat face has a radius of curvature that is infinite, and the general roller and cylindrical followers have a finite (but nonzero) radius of curvature. In this discussion, only planar cams will be considered, so no distinction between spherical and cylindrical follower faces will be made. Also, if only geometric information is of interest, no distinction needs to be made

8.3 SYNTHESIS OF MOTION PROGRAMS

The problem of motion-program synthesis is the problem of filling in, in an optimal way, the portions of the motion cycle that are not completely specified. The characteristics of the problem may be demonstrated by consideration of a cam that is required to drive a follower that dwells at 0 for a cam rotation of 60°, dwells at 1.0 in for a cam rotation of 110° to 150°, and is required to move with constant velocity from a displacement of 0.8 to 0.2 in for 200° to 300° of cam rotation. The specified portions of the motion program are displayed in Fig. 8.5.

A simple solution to the problem of filling the gaps is simply to move the cam at constant velocity between the specified segments, giving a follower displacement diagram as shown in Fig. 8.6.

Notice, however, that if this is done, the velocity is discontinuous at cam angles 60°, 110°, 150°, 200°, 300°, and 360°, causing the acceleration to become infinite at these locations. Since the follower cannot follow an infinite acceleration, this leads to loss of contact and/or excessive local stresses and resultant noise and wear problems.

![Figure 8.5](image1.png)

**FIGURE 8.5** The statement of the required displacements of a cam design problem in graphical form.

![Figure 8.6](image2.png)

**FIGURE 8.6** A cam angle—follower displacement program that satisfies the displacement requirements specified in Fig. 8.5.

The preceding motion program matches only the displacements at the ends of the segments. The infinite acceleration problem can be removed by matching both displacement and velocity at the ends of segments of the program. One way to do this is to subdivide the synthesized segments into two parts with a constant acceleration on the first and constant deceleration on the second. On such a subsegment, if the acceleration is \(a\), the velocity is given by

\[
v = v_0 + at
\]

where \(v_0\) is the velocity at the beginning of the segment. The displacement is given by

\[
y = s_0 + v_0t + \frac{a}{2}t^2
\]
where \( s_0 \) is the displacement at the beginning of the segment. Now, if the cam is driven at constant velocity,

\[ \theta = \theta_0 + \omega t \]

where \( \theta \) is the cam angle, \( \theta_0 \) is the cam angle at the beginning of the segment, and \( \omega \) is the angular velocity. Hence

\[ t = \frac{(\theta - \theta_0)}{\omega} \]

\[ v = v_0 + \frac{(\theta - \theta_0)}{\omega} \]

\[ y = s_0 + v_0 \frac{(\theta - \theta_0)}{\omega} + \frac{(\theta - \theta_0)^2}{2\omega^2} \]

Therefore, the relationship between \( s \) and \( \theta \), as plotted on the follower-displacement diagram, is parabolic (see Fig. 8.7). Cam–follower displacement programs that use this type of transition are called parabolic. The cam profiles developed from them are also called “parabolic.” It is important to understand that a so-called parabolic cam does not have a parabolic curve in its profile. Rather, the parabolas are in the transition curves used in the follower-displacement program.

**8.4 FOLLOWER DISPACEMENT FUNCTIONS**

Several different standard functions can be used to connect the parts of the displacement diagram where a specific type of motion is required. These displacement profiles ultimately determine the shape of the cam. Many different types of motions have been used in practice, and some have been extensively studied. These include the following:

1. uniform motion,
2. parabolic motion,
3. simple harmonic motion,
4. cycloidal motion, and
5. general polynomial motion.

The first two types of program have already been introduced. The first four types of program can be generated graphically as well as analytically, but the fifth type is generated only analytically. Both graphical and analytical development will be considered here, where possible. Both methods assume that the angular velocity, \( \omega \), of the cam is constant. If this is the case, then

\[ y = y(\theta) \]

And

\[ \theta = \theta_0 + \omega t \]
Here, $y$ is used as a generic output variable. It may correspond to either a linear or angular displacement of the follower. Note that if the cam motion is given as a function of time, the motion can easily be represented as a function of the cam rotation in degrees using the preceding expressions.

The higher derivatives are given by

$$\ddot{y} = \frac{d^2 y}{dt^2} = \frac{dy}{d\theta} \frac{d\theta}{dt} = y' \omega$$

and

$$\dddot{y} = \frac{d^3 y}{dt^3} = \frac{d}{dt} \left( \frac{dy}{d\theta} \frac{d\theta}{dt} \right) = \frac{d^2 y}{d\theta^2} \left( \frac{d\theta}{dt} \right)^2 + \frac{dy}{d\theta} \frac{d^2 \theta}{dt^2} = \frac{d^2 y}{d\theta^2} \omega^2 + \frac{dy}{d\theta} \alpha$$

But because $\omega$ is constant, $\alpha = 0$ and

$$\dddot{y} = y'' \omega^2$$

Therefore, $\dot{y}$ is a simple constant times $y'$, and $\ddot{y}$ is also a constant times $y''$. Consequently, even though we ultimately want to know the response to the time derivatives ($\dot{y}$ and $\ddot{y}$), we may work directly with the derivatives ($y'$ and $y''$) with respect to the cam displacement. If the cam velocity is not a constant, then the cam profile can be designed for only one operating situation if higher derivatives are important. In the following, a constant-velocity cam is assumed, and $y$ is again used to represent either an angular or linear displacement of the follower. Similarly, $\theta$ is used for the displacement of the cam, and it may be either an angular or a linear displacement.

The follower curves can be studied in terms of the simple diagram shown in Fig. 8.8. A general displacement diagram will be made up of three or more parts:

1. rises (1 or more),
2. returns (1 or more), and
3. dwells (0 or more).

Both the rise and return parts will contain one or more inflection points. These are points where a maximum slope is reached, and they correspond to points on the cam surface with maximum steepness. These points are identified by the locations where the curvature of the diagram changes sign. At the inflection points, the radius of curvature of the curve is infinite.

In each of the standard curve cases, we will look at mainly the rise part of the follower profile. The return part can be determined using the mirror images of the curves considered.

![FIGURE 8.8 Terminology used when discussing follower-displacement programs.](image)

### 8.5 UNIFORM MOTION

Uniform motion is represented in Fig. 8.9. To derive the equations for the follower displacement, a general form for the mathematical expression corresponding to the type of motion is assumed. The general equation will have undetermined constants in it, and these constants can be determined by matching boundary conditions at the two ends of the curve. For uniform motion, the general form of the curve used is
If \( L \) is the amount of the rise, and \( \beta \) is the cam rotation required for the rise, then the constant \( C \) must be \( L/\beta \) and \( y \) becomes

\[
y = \frac{L}{\beta} \theta
\]

During the rise, the velocity and acceleration are

\[
\dot{y} = \frac{L}{\beta} \omega
\]

\[
\ddot{y} = 0
\]

The displacement, velocity, and acceleration are plotted in Fig. 8.10. As noted earlier, the acceleration is infinite at the points where the uniform motion meets the dwells. Therefore, even for low speeds and elastic members, the forces transmitted will be very large. For very low speeds, however, this type of displacement diagram might be acceptable.

Graphically, the uniform motion–displacement diagram is characterized by a uniform change in \( y \) for a uniform change in the cam motion. This condition is shown in Fig. 8.9.

8.6 PARABOLIC MOTION

The equations for parabolic motion can be derived using the same procedure as described in Section 8.3. However, two parabolas must be used for each transition. The two parabolas meet at the point midway between the ends of the two dwell regions. The general form for both parabolas is

\[
y = C_0 + C_1 \theta + C_2 \theta^2
\]
\[ y' = C_1 + 2C_2 \theta \]
\[ y'' = 2C_2 \]

(8.2)

If the cam displacement is taken as 0 at the beginning of the rise, then at \( \theta = 0 \),
\( y = y' = 0 \). Then, \( C_0 = C_1 = 0 \). Also, at \( \theta = \beta/2 \), \( y = L/2 \). Therefore, the displacement and first
and second derivatives with respect to \( \theta \) are

\[ y = 2L \left( \frac{\theta}{\beta} \right)^2 \]
\[ y' = 4 \frac{L}{\beta^2} \theta \]
\[ y'' = 4 \frac{L}{\beta^2} \]

(8.3)

and the velocity and acceleration are

\[ \dot{y} = 4 \frac{L \omega}{\beta^2} \theta \]
\[ \ddot{y} = 4 \frac{L \omega^2}{\beta^2} \]

At the point at which the curve meets the first dwell, the velocity and acceleration are
continuous, but the third derivative, or jerk, is infinite. This derivative is proportional to
the change in force and for high-speed cams is an important aspect of the motion. Although not
so serious as having an infinite acceleration pulse, an infinite jerk can excite vibratory
behavior in the system.

For the second half of the rise, the conditions to match are at \( \theta = \beta/2, y = L/2 \), and at
\( \theta = \beta, y = L \), and \( y' = 0 \). Then from Eqs. (8.61) and (8.62), we get

\[ \frac{L}{2} = C_0 + C_1 \frac{\beta}{2} + C_2 \left( \frac{\beta}{2} \right)^2 \]
\[ L = C_0 + C_1 \beta + C_2 \beta^2 \]
\[ 0 = C_1 + 2C_2 \beta \]

The solution to this linear set of equations yields

\[ C_0 = -L \]
\[ C_1 = \frac{4L}{\beta} \]
\[ C_2 = -\frac{2L}{\beta^2} \]

So that

\[ y = L \left[ 1 - 2 \left( 1 - \frac{\theta}{\beta} \right)^2 \right] \]

(8.4)

And

\[ y' = \frac{4L}{\beta} \left( 1 - \frac{\theta}{\beta} \right) \]
\[ y'' = -\frac{4L}{\beta^2} \]

Finally, the velocity, acceleration, and jerk are given by
\[
y = \frac{4Lo}{\beta} \left( 1 - \frac{\theta}{\beta} \right)
\]
\[
j = -\frac{4Lo^2}{\beta^2}
\]

These equations apply to the segment of the program to the right of the inflection point shown in Fig. 8.11.

Graphically, the part of the curve up to the inflection point can be generated using the construction shown in Fig. 8.12. For the construction, the horizontal axis is divided into uniform increments, and the maximum rise is evenly divided into the same number of equal increments. The point at the origin is then connected to each of the points on the line of the maximum rise. Points on the displacement curve are given by the intersection of the diagonal lines with the corresponding vertical lines.

A cam return using parabolic motion is shown in Fig. 8.13. To determine the equations for the return from \( y = L \) to 0 during the angular displacement \( \beta \), we can use Eq. (8.1) again but with different boundary conditions. To simplify the equations, we will shift the origin of the coordinate system to the end of the dwell at the beginning of the return. For the first part of the return, \( y = L \) and \( y' = 0 \) at \( \theta = 0 \) and \( y = L/2 \) at \( \theta = \beta/2 \). For these conditions, \( C_0 = L, C_1 = 0, \) and \( C_2 = \frac{2L}{\beta^2} \).

The displacement equation is
\[
y = L \left[ 1 - 2 \left( \frac{\theta}{\beta} \right)^2 \right]
\]
(8.5)

For the second half of the return, the conditions to match are at \( \theta = \beta/2, y = L/2 \), and at \( \theta = \beta, y = 0 \), and \( y' = 0 \). For these conditions,
\[
y = 2L \left( 1 - \frac{\theta}{\beta} \right)^2
\]
(8.6)

**FIGURE 8.11** Displacement, velocity, acceleration, and jerk relations for parabolic motion during rise.

**FIGURE 8.12** Construction of parabolic segment of follower-displacement program.

**FIGURE 8.13** Displacement, velocity, acceleration, and jerk relations for parabolic motion during return.
In general, the rise and return will not always start at $\theta = 0$. However, in these cases, a simple coordinate transformation can be used. If the rise or return actually starts at $\theta = \gamma$, substitute $(\theta - \gamma)$ wherever $\theta$ appears in Eqs. (8.3)–(8.6).

**EXAMPLE 8.1**  
**Design for Parabolic Motion**

Design a parabolic cam–follower displacement program to provide a dwell at zero lift for the first 120° of the motion cycle and to dwell at 0.8 in lift for cam angles from 180° to 210°. The cam profile will be laid out using 10° plotting intervals. Assume that the cam rotates with constant angular velocity.

The motion specification is as shown in Fig. 8.14. For the first part of the rise ending at $\phi = 150°$ in the interval 120° to 180°, Eq. (8.3) applies if we use $\theta = (\phi - 120°)$ and 0.8 = $L$. The resulting expression for the first part of the rise is

$$
y = 2L \left( \frac{\theta}{\beta} \right)^2 = 1.6 \left( \frac{\phi - 120°}{60°} \right)^2
$$

(8.7)

![Graph of follower displacement vs. cam angle](image)

**FIGURE 8.14**  
The motion specification for Example 8.1.

For the second part of the rise starting at $\theta = 150°$, Eq. (8.4) applies if we use $\theta = (\phi - 120°)$ and 0.8 = $L$. The resulting expression is

$$
y = L \left[ 1 - 2 \left( 1 - \frac{\theta}{\beta} \right) \right] = 0.8 \left[ 1 - 2 \left( 1 - \left( \frac{\phi - 120°}{60°} \right) \right)^2 \right]
$$

(8.8)

Using Eqs. (8.7) and (8.8) produces the successive lifts given in Table 8.1. For the first part of the return ending at $\theta = 285°$ in the interval 210° to 360°, Eq. (8.5) applies if we use $\theta = (\phi - 210°)$ and 0.8 = $L$. The resulting expression for the first part of the return is

$$
y = L \left[ 1 - 2 \left( \frac{\theta}{\beta} \right) \right] = 0.8 \left[ 1 - 2 \left( \frac{\phi - 210°}{150°} \right) \right]^2
$$

(8.9)

For the second part of the return starting at $\theta = 285°$, Eq. (8.6) applies if we use $\theta = (\phi - 210°)$ and 0.8 = $L$. The resulting expression is

$$
y = 2L \left( 1 - \frac{\theta}{\beta} \right)^2 = 1.6 \left( 1 - \frac{\phi - 210°}{150°} \right)
$$

(8.10)

Using Eqs. (8.9) and (8.10) produces points on the return curve given in Table 8.2. The resulting transition curves are plotted in Fig. 8.15. Notice that the lift values are tabulated to four decimal places. Cam and follower systems normally use very rigid components and even small profile variations are important. For this reason, we normally work with at least four decimal places when doing cam calculations. Gears are another type of profile mechanism in which the components are very rigid and, consequently, even tiny profile variations can be important.

**TABLE 8.1**  
Cam–Follower Data for Rise in Example 8.1

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>120°</th>
<th>130°</th>
<th>140°</th>
<th>150°</th>
<th>160°</th>
<th>170°</th>
<th>180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.0000</td>
<td>0.0444</td>
<td>0.1778</td>
<td>0.4000</td>
<td>0.6222</td>
<td>0.7556</td>
<td>0.8000</td>
</tr>
</tbody>
</table>
8.10 DETERMINING THE CAM PROFILE

Once the follower motion is determined as a function of the cam displacement, the cam surface can be found either graphically or analytically. For extremely accurate cams, the geometry must be determined analytically and the machining must be done using CNC milling machines. For low-speed cams, however, a graphical layout and manual machining are adequate.

In both the graphical and analytical approaches to determining the cam geometry, the cam mechanism must be inverted. That is, the cam is taken as the reference system, and the frame and follower are considered to move relative to the cam. To maintain the correct relative motion, the follower will move relative to the cam in a direction opposite to the motion of the cam relative to the follower.

If we restrict our discussions to planar, rotating cams, four general types of followers are possible: (a) a translating cylindrical-faced follower, (b) a translating flat-faced follower, (c) a rotating cylindrical-faced follower, and (d) a rotating flat-faced follower (Fig. 8.22).

Notice that the cam geometry is independent of the type of joint between the cylindrical-faced follower and the cam. The kinematic design procedure is exactly the same when a roller follower or a solid cylindrical-faced follower is involved. We will consider both graphical and analytical approaches to the design of the cam for each type of follower shown in Fig. 8.22.

8.10.1 Graphical Cam Profile Layout

As already indicated, cam profiles are laid out graphically using inversion. That is, the cam is viewed as stationary, and the successive positions of the follower are located relative to it. This results in a polar plot of successive follower positions. The cam profile is then filled in as the envelope curve of the follower positions.

The first step in laying out the cam profile is to select a base circle radius. The base circle represents the position of the follower at zero lift. Successive lift values are plotted radially outward from the base circle.

Choosing a large base circle radius results in a large cam. However, if the base circle is too small the cam profile may have hollows of smaller radius than the follower. Since the follower will bridge across such a hollow, it will not follow the desired lift program. Obviously, this situation must be avoided, and it is therefore necessary to have a means of computing the radius of curvature of the cam at different locations.

The pressure angle of a cam is the angle between the contact normal and the velocity of the point on the follower at the contact location. Reducing the pressure angle reduces the contact loads and promotes smoother operation with less wear. Increasing the base circle radius decreases the maximum value of pressure angle. Thus, it is good practice to use the largest base circle that the design constraints will allow. As a general rule of thumb, the base circle radius should be two to three times the maximum lift value.
EXAMPLE 8.3
Layout of a Cam Profile for a Radial Roller Follower

Lay out a cam profile using the harmonic follower displacement profile of Example 8.2. That is, the follower is to dwell at zero lift for the first 120° of the motion cycle and to dwell at 0.8 in lift for cam angles from 180° to 210°. The cam is to have a translating, roller follower with a 1-in roller diameter. The cam will rotate clockwise. Lay out the cam profile using 10° plotting intervals.

Solution

The basic motion specification is as shown in Fig. 8.15. Using the results of Example 8.2 produces the lift values to be plotted given in Table 8.6. Notice that the dwells correspond to locations on the cam where the radius is constant.

| Table 8.6 Follower Displacements for Example 8.3 |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| θ (°)           | 0               | 30              | 60              | 90              | 120             | 150             | 180             | 210             | 240             | 270             | 300             | 330             | 360             | 390             | 420             | 450             | 480             | 510             | 540             | 570             | 600             |
| γ (°)           | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          |
| ϕ (°)           | 90              | 120             | 150             | 180             | 210             | 240             | 270             | 300             | 330             | 360             | 390             | 420             | 450             | 480             | 510             | 540             | 570             | 600             |
| β (°)           | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0000          | 0.0536          | 0.2000          | 0.4000          | 0.6000          | 0.7464          | 0.2000          | 0.4000          | 0.6000          | 0.7464          | 0.2000          | 0.4000          | 0.6000          | 0.7464          | 0.2000          | 0.4000          | 0.6000          |
| ψ (°)           | 0.8000          | 0.8000          | 0.8000          | 0.8000          | 0.7913          | 0.7654          | 0.7236          | 0.6677          | 0.6000          | 0.5236          | 0.4418          | 0.3582          | 0.2764          | 0.2000          | 0.1323          | 0.0764          | 0.0346          | 0.0087          | 0.0000          | 0.0000          |

FIGURE 8.23  Layout of the cam profile for Example 8.3. The process of laying out a cam profile is one of inversion. That is, the cam is viewed as being stationary, and successive positions of the follower are plotted relative to it. In this case, a prime circle of 2.0-in radius was chosen. This represents the location of the follower center at zero lift. Positive lift values are plotted outward from the base circle. The successive positions of the follower are then drawn using the plotted points as centers. Finally, the profile is plotted as an envelope curve of the successive follower positions. Because of the inversion, if the cam is to rotate clockwise, the positions of the follower must be plotted in the opposite direction, that is, counterclockwise.
The layout of the cam is accomplished by drawing radial lines at 10° increments. Because the cam rotates clockwise, the radial lines are laid off and labeled in the counterclockwise direction, as shown in Fig. 8.23. Next, the base circle and the prime circle are drawn. The base circle is chosen to have a 1.5-in radius, and it is the largest circle that can be drawn inside the cam profile and be tangent to the cam profile. The radius of the prime circle is equal to \( r_b / r_0 \) where \( r_b \) is the base circle radius and \( r_0 \) is the radius of the roller follower. In this problem, the prime-circle radius is 2.0 in. The cam profile is initially laid off from the prime circle to give the pitch curve. The pitch curve is the curve traced by the center of the roller follower. Notice that the pitch curve will be the cam profile if \( r_0 \) is zero. This corresponds to the case of a knife-edged follower.

Once the radial lines and prime circle are established, the displacements can be laid off from the prime circle as shown in Fig. 8.23. The radius of the follower is drawn with its center located on the pitch curve at a series of locations. The cam can be defined by drawing a curve tangent to roller locations as shown in Fig. 8.23.

As indicated before, an important parameter for cam motion is the pressure angle. In the case of the translating, roller follower, this is the angle between the direction of the follower travel and the normal to the curve. For a given force on the follower roller, the force in the direction of travel of the follower will be proportional to the cosine of the pressure angle. The force normal to the travel of the follower is proportional to the sine of the pressure angle. Wear on the follower stem will increase with the normal force. Therefore, from design considerations, we want the pressure angle (\( \alpha \)) to be as small as possible.

The maximum pressure angle will occur at the pitch points. These correspond to the inflection points on the follower displacement curves (see Fig. 8.8). If the torque on the cam is more or less constant, the pressure angles at the pitch points will correspond to the parts of the cycle where the maximum normal force occurs and hence the times when the follower stem wear will be greatest. It will also correspond to the parts of the cycle where the follower will tend to bind in the stem bearing. Because of problems with wear and binding, the pressure angle is usually limited to angles on the order of \( \pm 30° \). If the pressure angle becomes excessive, the base circle should be increased or the follower-displacement profile should be changed.

The problem statement indicated that a roller follower was to be designed. However, the construction would be exactly the same if a solid cylindrical-faced follower were involved. From the standpoint of the cam geometry, the important issues are the radius of the cylindrical face and the direction of translation relative to the cam.

EXAMPLE 8.4

Layout of a Cam Profile for a Radial Flat-Faced Follower

Again, lay out a cam profile using the harmonic displacement profile of Examples 8.2 and 8.3. The cam is to have a translating flat-faced follower that is offset by 0.2 in. The cam will rotate clockwise. Lay out the cam profile using 10° plotting intervals.

Solution

The basic motion specification is the same as in Example 8.3 (Table 8.6). The layout of the cam is again accomplished by drawing radial lines at 10° increments. Because the cam rotates clockwise, the radial lines are laid off and labeled in the counterclockwise direction as was done in Fig. 8.23. Next the base circle is drawn. Because a flat-faced follower is being designed, there is no prime circle. However, selection of the base circle requires careful consideration.

A major restriction on the cam profile driving a flat-faced follower is that the profile must form a convex surface. This means that the vectors from every point on the cam to the corresponding center of curvature must point toward the interior of the cam. An alternative way to approach the convexity problem is to imagine an arbitrary line drawn across the face of the cam. If it is possible to select an arbitrary line that intersects the cam at more than two points, the cam profile is not convex. If the cam is not convex, the flat-faced follower cannot contact the cam at all points, and the desired motion will not be generated. This condition will be illustrated mathematically when an analytical approach to cam synthesis is discussed. Clearly, the cam generated in Fig. 8.23 does not satisfy the convexity condition; however, this is not necessarily an issue with roller followers. When the resulting cam is not convex for flat-faced followers, we must increase the size of the base circle or change the follower-displacement function. The effect of changing the base circle can be easily investigated by running one of the cam design programs supplied on the disk with this book.

To begin the construction, we can select a base circle somewhat arbitrarily. However, if the radius of curvature at some location on the resulting cam is too small, the base circle diameter must be increased.

The follower is offset, but this does not affect the geometry of the cam. All points on the follower have the same velocity because its motion is pure translation. Therefore, from a kinematic standpoint, the actual location of the follower stem is not important. From a machine design standpoint it is important, however, because the larger the offset, the higher the moment on the follower stem.
We can lay off the displacements in Table 8.6 from the base circle and along the radial lines. At each of these locations on the radial lines, draw a line perpendicular to each radial line. These perpendicular lines correspond to the face of the follower. This is illustrated in Fig. 8.24. The lines for different positions of the follower will form an envelope that defines the geometry of the cam surface. We construct the outline of the cam by drawing a curve that contacts the lines corresponding to the different positions of the follower face at the tangent points.

![Diagram of cam and follower relationship](image)

**FIGURE 8.24** Basic construction lines for determining the cam profile for a flat-faced follower.

As the lines corresponding to the different positions of the follower face are drawn, successive lines will intersect. For the geometry to be valid, the angle increment for successive intersections must be positive. If an intersection requires a negative angle increment, it will not be possible to generate the cam, and a larger base circle must be used. This situation is illustrated in the current problem in Fig. 8.25 for the positions corresponding to rotation angles of 150°, 160°, 170°, and 180°. In Fig. 8.25, a base circle radius of 1.5 in was chosen. The angle increment for the intersection corresponding to 160° and 170° is positive, but the increment corresponding to 170° and 180° is negative. This situation makes the cam nonconvex and indicates that the base circle is too small and must be increased.

![Diagram showing conditions for cam generation](image)

**FIGURE 8.25** Condition when the base circle is too small to generate an acceptable cam for a flat-faced follower.
The smallest base circle is the one for which the angle increment corresponding to 170° and 180° is no longer negative, that is, when it is zero. This occurs when the follower-face lines corresponding to 160°, 170°, and 180° intersect. This occurs for a base circle radius of approximately 5.5 in. For this base circle, the cam will have a point or cusp corresponding to the location where the face lines intersect. The envelope of the face lines and the resulting cam is shown in Fig. 8.26. The cam designed for the roller follower and for the same displacement profile is also shown in Fig. 8.26 for comparison. Based on the size of the cam required, a flat-faced follower would not be a good choice for this type of displacement profile.

After the follower-face locations are found, the stem locations can be shown by drawing parallel lines to the radial lines. These are also shown in Fig. 8.26.

As indicated before, an important parameter for cam motion is the pressure angle. When a flat-faced follower is used, the normal to the follower profile is always in the direction of the follower travel if the follower face is perpendicular to the stem. This makes the pressure angle always zero; however, there can be significant lateral loads on the follower bearings caused by the frictional force at the cam–follower interface and by the moment generated by the normal force at the cam–follower interface and the offset line of action. The friction force can be reduced by lubrication but never completely eliminated, and the bearing couple that opposes the moment from the normal force must be addressed in the design of the cam and follower system. Depending on the lubrication and design, the lateral forces can be as high as or higher than the corresponding lateral force with a roller follower. Also, the cam may be so large that, to avoid the convexity condition, the roller follower would be preferred from size considerations.

Another important parameter that must be determined for the design of a cam for a flat-faced follower is the size (length) of the follower face. It is essential that the face be long enough on both sides to maintain contact with the cam on a tangent line. The minimum length of the follower face can be established by direct measurement. The actual length would be equal to the minimum length plus a small increment.
PROBLEMS

EXERCISE PROBLEMS ON FOLLOWER-DISPLACEMENT SCHEDULES

8.1 A cam that is designed for cycloidal motion drives a flat-faced follower. During the rise, the follower displaces 1 in for 180° of cam rotation. If the cam angular velocity is constant at 100 rpm, determine the displacement, velocity, and acceleration of the follower at a cam angle of 60°.

8.2 A constant-velocity cam is designed for simple harmonic motion. If the flat-faced follower displaces 2 in for 180° of cam rotation and the cam angular velocity is 100 rpm, determine the displacement, velocity, and acceleration when the cam angle is 45°.

8.3 A cam drives a radial, knife-edged follower through a 1.5-in rise in 180° of cycloidal motion. Give the displacement at 60° and 100°. If this cam is rotating at 200 rpm, what are the velocity (ds/dt) and the acceleration (d²s/dt²) at θ = 60°?

8.4 Draw the displacement schedule for a follower that rises through a total displacement of 1.5 in with constant acceleration for 1/4 revolution, constant velocity for 1/4 revolution, and constant deceleration for 1/4 revolution of the cam. The cam then dwells for 1/8 revolution, and returns with simple harmonic motion in 1/4 revolution of the cam.

8.5 Draw the displacement schedule for a follower that rises through a total displacement of 20 mm with constant acceleration for 1/8 revolution, constant velocity for 1/4 revolution, and constant deceleration for 1/4 revolution of the cam. The cam then dwells for 1/4 revolution and returns with simple harmonic motion in 1/4 revolution of the cam.

8.6 Draw the displacement schedule for a follower that rises through a total displacement of 30 mm with constant acceleration for 90° of rotation and constant deceleration for 45° of cam rotation. The follower returns 15 mm with simple harmonic motion in 90° of cam rotation and dwells for 45° of cam rotation. It then returns the remaining 15 mm with simple harmonic motion during the remaining 90° of cam rotation.

8.7 Draw the displacement schedule for a follower that rises through a total displacement of 3 in with cycloidal motion in 120° of cam rotation. The follower then dwells for 90° and returns to 0° with simple harmonic motion in 90° of cam rotation. The follower then dwells for 60° before repeating the cycle.

8.8 A cam returns from a full lift of 1.2 in during its initial 60° rotation. The first 0.4 in of the return is half-cycloidal. This is followed by a half-harmonic return. Determine β₁ and β₂ so that the motion has continuous first and second derivatives. Draw a freehand sketch of y', y'', and y''' indicating any possible mismatch in the third derivative.

8.9 Assume that s is the cam–follower displacement and θ is the cam rotation. The rise is 1.0 cm after 1.0 rad of rotation, and the rise begins and ends at a dwell. The displacement equation for the follower during the rise period is

\[ s = h \sum_{i=0}^{n} C_i \left( \frac{\theta}{\beta} \right)^i \]

If the position, velocity, and acceleration are continuous at \( \theta = 0 \), and the position and velocity are continuous at \( \theta = 1.0 \) rad, determine the value of \( n \) required in the equation, and find the coefficients \( C_i \) if \( \theta = 2 \) rad/s. Note: Use the minimum possible number of terms.

8.10 Re-solve Problem 8.9 if \( \theta = 0.8 \) rad and \( \dot{\theta} = 200 \) rad/s.

8.11 For the cam-displacement schedule given, h is the rise, \( \beta \) is the angle through which the rise takes place, and s is the displacement at any given angle \( \theta \). The displacement equation for the follower during the rise period is

\[ s = h \sum_{i=0}^{n} a_i \left( \frac{\theta}{\beta} \right)^i \]

Determine the required values for \( a_0, \ldots, a_i \) such that the displacement, velocity, and acceleration functions are continuous at the end points of the rise portion.
8.12 Re-solve Problem 8.11 if \( h = 20 \text{ mm} \) and \( \beta = 120^\circ \).
8.13 Re-solve Problem 8.11 if \( h = 2 \text{ in} \) and \( \beta = 90^\circ \).
8.14 Assume that \( s \) is the cam–follower displacement and \( \theta \) is the cam rotation. The rise is \( h \) after \( \beta \) degrees of rotation, and the rise begins at a dwell and ends with a constant velocity segment. The displacement equation for the follower during the rise period is

\[
s = h \sum_{n=0}^{\beta} C_n \left( \frac{\theta}{\beta} \right)^n
\]

If the position, velocity, and acceleration are continuous at \( \theta = 0 \) and the position and velocity are continuous at \( \theta = \beta \), determine the \( n \) required in the equation, and find the coefficients \( C_n \) that will satisfy the requirements if \( s = h = 1.0 \).

8.15 A follower moves with simple harmonic motion a distance of 20 mm in 45° of cam rotation. The follower then moves 20 mm more with cycloidal motion to complete its rise. The follower then dwells and returns 25 mm with cycloidal motion and then moves the remaining 15 mm with harmonic motion in 45°. Find the intervals of cam rotation for the cycloidal motions and dwell by matching velocities and accelerations, then determine the equations for the displacement \( s \) as a function of \( \theta \) for the entire motion cycle.

**EXERCISE PROBLEMS ON GRAPHICAL CAM DESIGN**

8.16 Construct the part of the profile of a disk cam that follows the displacement diagram shown. The cam has a 5-cm-diameter pitch circle and is rotating counterclockwise. The follower is a knife-edged, radial, translating follower. Use 10° increments for the construction.

8.17 Construct the profile of a disk cam that follows the displacement diagram shown. The follower is a radial roller and has a diameter of 10 mm. The base circle diameter of the cam is to be 40 mm and the cam rotates clockwise.

8.18 Accurately sketch one-half of the cam profile (stations 0–6) for the cam follower, base circle, and displacement diagram given. The base circle diameter is 1.2 in.

8.19 Lay out a cam profile using a harmonic follower displacement (both rise and return). Assume that the cam is to dwell at zero lift for the first 100° of the motion cycle and to dwell at 1 in lift for cam angles from 160° to 210°. The cam is to have a translating, radial, roller follower with a 1 in roller diameter, and the base circle radius is to be 1.5 in. The cam will rotate clockwise. Lay out the cam profile using 20° plotting intervals.

8.20 Lay out a cam profile using a cycloidal follower displacement (both rise and return) if the cam is to dwell at zero lift for the first 80° of the motion cycle and to dwell at 2 in lift for cam angles from 120° to 190°. The cam is to have a translating, radial, roller follower with a roller diameter of 0.8 in. The cam will rotate counterclockwise, and the base circle diameter is 2 in. Lay out the cam profile using 20° plotting intervals.

8.21 Lay out a cam profile assuming that an oscillating roller follower starts from a dwell for 0° to 140° of cam rotation, and the cam rotates clockwise. The rise occurs with parabolic motion during the cam rotation from 140° to 220°. The follower then dwells for 40° of cam rotation, and the return occurs with parabolic motion for the cam rotation from 260° to 360°. The amplitude of the follower rotation is 35°, and the follower radius is 1 in. The base circle radius is 2 in, and the distance between the cam axis and follower rotation axis is 4 in. Lay out the cam pro-