CHAPTER SIX
PRODUCTION THEORY AND ESTIMATION

The Production Function:
• The production function is the technical relationship, which shows the maximum attainable production levels using different combinations of input at a given state of technology within a given period of time.

Mathematically, the production function can be expressed as

\[ Q = f(L, K, N, T, \ldots) \]

• \( Q \) is level of output produced;
• \( L \) is the number of workers (including entrepreneurship);
• \( K \) is the capital;
• \( N \) is land;
• \( T \) is the state of technology; and
• \( \ldots \) refers to other inputs used in the production process

For simplicity we will consider a production function of two inputs: labor and capital.
• Labor and capital are both composite inputs that include all other factors of production, so, the production function is normally written in the follows implicit form:

\[ Q = f(L, K); f_L > 0, f_{LL} < 0; f_K > 0, f_{KK} < 0 \]

• The above equation tells us that the first partial derivatives of output, with respect to each of the inputs, are positive, i.e., \( Q \) has a positive relationship with \( L \) and \( K \).
• The second partial derivatives of output with respect to each of the inputs are negative, indicating that the production function has some maximum points.

The relationship between inputs and output assumes:
1. Fixed state of technology
2. Efficient use of input combinations
3. Given time period
Production depends on the time-frame in which the firm is operating (short run and long run).

The short-run production function shows the maximum quantity of good or service that can be produced by a set of inputs when at least one of the inputs used remains unchanged.

The long-run production function shows the maximum quantity of good or service that can be produced by a set of inputs, assuming the firm is free to vary the amount of all the inputs being used.

The relationship between output and the quantity of labor employed, assuming capital is fixed, can be described using the following three concepts:

1. Total product
2. Marginal product
3. Average product

Total Product:

- Total product is the total output produced in a given period.
- Because of our assumption for simplicity, the production function depends only on labor and capital.
- Hence, to increase output in the short run, a firm must increase the amount of labor employed.

The total product curve shows how total product changes with the quantity of labor employed.

The area below TP curve is attainable at every level of labor, while the area above TP is unattainable.

Example:

- A small farm with fixed capital (area of the farm, water well, and equipments), and variable number of workers.
- Under these set of assumptions, total production (TP), marginal Product (MP), and average product (AP), may be represented by the following hypothetical values:

<table>
<thead>
<tr>
<th>Number of Workers (L)</th>
<th>Total Product (TP)</th>
<th>Marginal Product of labor (MP_L)</th>
<th>Average product of labor (AP_L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>------</td>
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</tr>
<tr>
<td>1</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>29</td>
<td>11</td>
<td>9.67</td>
</tr>
<tr>
<td>4</td>
<td>39</td>
<td>10</td>
<td>9.75</td>
</tr>
<tr>
<td>5</td>
<td>47</td>
<td>8</td>
<td>9.4</td>
</tr>
<tr>
<td>6</td>
<td>52</td>
<td>5</td>
<td>8.67</td>
</tr>
<tr>
<td>7</td>
<td>56</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>8</td>
<td>52</td>
<td>-4</td>
<td>6.5</td>
</tr>
</tbody>
</table>
From the table above, you may notice that the total product in the short run:
1. increases first at an increasing rate
2. Until it reaches an infliction (turning) point,
3. after that it keeps increasing but at a decreasing rate until it reaches its maximum level,
4. then it starts to fall.

The following graph reflects these phases of production:

The Marginal Product:
- The marginal product of labor (MPL) is the change in output (or Total Product) that results from a one-unit change in the variable input (quantity of labor employed), or one additional hour of work, with all other inputs remaining the same.

\[ \text{MPL} = \frac{\Delta \text{TP}}{\Delta L} = \frac{\Delta Q}{\Delta L} \]

Graphically, MPL measures the slope of the total product curve.

As you see in the graph above,
- the slope of the TP curve starts very close to zero at the origin, then increases (production speeds up) to reach its maximum at the infliction (turning) point (point a) on the TP curve, where MPL curve reaches its maximum.
- The slope of TP curve decreases gradually (production slows down) to reach zero at L2 as MPL falls to intersects the L axis.
- Total product start to fall from there on as the slope of the MPL curve become negative.

From the above table notice how the Marginal product of labor increases to reach its maximum of 11, then falls after that to take a negative value where total product starts to fall.

Almost every production process has two features: initially, increasing marginal returns (IMR), and eventually diminishing marginal returns (DMR).
2. Then, **diminishing marginal returns**
   - Diminishing marginal returns arises from the fact that employing additional units of labor means each worker has less access to capital and less space in which to work.
   - Diminishing marginal returns starts to take effect when MP starts to decrease.
   - Diminishing marginal returns are so pervasive that they are elevated to the status of a “law.”

- The **law of diminishing returns** states that as a firm uses more of a variable input with a given quantity of fixed inputs, the marginal product of the variable input at some point will eventually falls, where each additional unit contribute less production than the preceded unit”.
- The law of diminishing returns is a short run concept that describes the change in the marginal product of the variable input.

- It is important to keep in mind that all units of the variable input are assumed of equal productivity, and the only reason for variations in its marginal product (productivity) can be attributed to its order in utilization in the production process.

- In general:
  1. When MP is $\uparrow$, firm is experiencing increasing marginal returns.
  2. When MP is $\downarrow$ but positive, firm is experiencing decreasing marginal returns.
  3. When MP is $\downarrow$ and negative, firm is experiencing negative marginal returns.

- **Example:**
  Given a production function: $Q = 15X - 5X^2$,
  - Find the level of input at which negative marginal returns starts
    $MP = \frac{dQ}{dX} = 15 - 10X$
    Negative returns starts when $MP = 0$
    Thus, set $MP = 0$ and solve for $X$
    $MP = 15 - 10X = 0 \Rightarrow X = 15/10 = 1.5$

- The **average product**:
  - The average product of labor ($AP_L$) refers to the share of each worker in the total production.
  - It is equal to total product divided by the quantity of labor employed. It is the total Product per unit of input used.

  $$AP_L = \frac{TP_L}{L} = \frac{Q}{L}$$
Example:
Given a production function:
\[ Q = 3X^2 - 5X^3 \]
then
\[ AP = \frac{Q}{X} = \frac{15X - 5X^2}{X} = 15 - 5X \]
Set AP = 15 - 5X = 0
\[ \Rightarrow X = \frac{15}{5} = 3 \]
(Notice that AP reach 0 at twice L as MP)

The relationship between TP and MP
1. Because of IMR, TP increases initially at increasing rate. Then, because of DMR, TP increases at decreasing rate.
2. The point that TP change its pace from increasing at increasing rate to increasing at decreasing rate (i.e., the point where DMR start its course) is called the turning point. When TP is at its turning point MPL is at its maximum.

The relationship between AP and MP
1. When MP > AP (MP Curve above AP Curve), then AP is rising.
2. When MP < AP (MP Curve below AP Curve), then AP is falling.
3. When MP = AP, (MP Curve intersect AP Curve) then AP is at its maximum.

The Three Stages of Production in The Short Run:
- Three stages can be noticed on the product curves presented in the above graph.

Stage I:
1. From zero units of the variable input to where AP is maximized (where \( AP_L = MPL \)).
2. At this stage, AP is rising.
3. In our example, stage I ends where four workers are being hired at \( L_1 \) on the figure above.
4. For a profit-maximizing firm, it is irrational to limit production to any level within the first stage.
5. The rising AP throughout this stage causes the average cost to fall and profits to rise as production expands.
### Stage II:
- From the maximum AP to where MP = 0 (or TP is maximum).
- At this stage, AP declining but MP is positive.
- In our example, Stage II is between $L_1$ and $L_2$ levels of employment.
- This is the rational stage of production, over which the firm manager has to figure out the optimal number of workers that maximizes profits.

### Stage III:
- From where MP=0 (or TP is maximum) onwards.
- At this stage, MP is negative or TP is declining.
- In our example, Stage III starts at $L_2$ ($MP_L = 0$) and above.
- Production in this stage is also irrational because the firm incurs higher costs to hire more workers; while the total revenue is falling as TP decreases.

### Why not Stage I?
- According to economic theory, in the short run, firms should operate in Stage II.
- Specialization and team work continue to add more output when additional variable input is used. Fixed input is being properly utilized \(\Rightarrow\) efficient use of resources.

### Why not Stage III?
- Firm uses more of its variable inputs to produce less output. Fixed input is over utilized or overused \(\Rightarrow\) inefficient use of resources.

### The Partial Elasticity of Production:
- In the short run, it is possible to measure the elasticity of production with respect to each of the variable inputs.
- In a similar way to the concept of demand elasticity, production elasticity measures the responsiveness of production to changes in one variable input when other inputs are held constant.
- It measures the percentage change in the quantity produced due to a one percent change in labor input.
It is known as the partial elasticity of production.

The elasticity of production with respect to labor may be presented in the following formula:

\[ E_L = \frac{\% \Delta Q}{\% \Delta L} = \frac{\frac{\Delta Q}{Q} \cdot \frac{L}{\Delta L}}{\frac{\Delta L}{L} \cdot \frac{Q}{\Delta Q}} = \frac{MP_L}{AP_L} \]

In the first stage, when \( MP_L > AP_L \), then the labor elasticity, \( E_L > 1 \).
- A one percent increase in labor will increase output by more than 1 percent.

In the second stage, when \( MP_L < AP_L \), the labor elasticity \( 0 < E_L < 1 \).
- A one percent increase in labor will increase output by less than 1 percent.

In the third stage of production, when \( MP_L \) is negative, \( E_L < 0 \).

Using the above form of partial elasticity of production, the firm manager, my easily find out the stage of his firm production.
- All he needs is estimating the production functions and the quantity of the major variable input used in production.
- The production elasticity of capital has the identical in form, except \( K \) appears in place of \( L \).

Example:
Consider the production function,
\[ Q = 100L - L^2, \]
find the stage of production if the firm uses 20 workers.

Solution
\[ MP_L = 100 - 2L = 100 - 2(20) = 60 \]
\[ AP_L = 100 - L = 100 - 20 = 80 \]
\[ E_L = \frac{MP_L}{AP_L} = \frac{60}{80} = 0.75. \]

Since \( E_L \) is positive and less than one, the firm is producing in stage II.

Example:
If \( Q = 9L^2 - L^3 \)
(Not that power 3 determines 3 stages)

a. Find the ranges of the 3 stages of production
b. Find at which stage the firm is operating if \( L = 5 \), and \( L = 3 \)
c. Find \( L \) value at the starting of DMRs
• Solution
a. Stage I: Between L = 0 and L where AP is maximized
\[
AP = \frac{Q}{L} = \frac{9L^2}{L} - \frac{L^3}{L} = 9L - L^2
\]
\[
dAP/dL = 9 - 2L = 0 \Rightarrow L = 9/2 = 4.5
\]
Stage I from 0 to 4.5

b. Suppose L = 5, at which stage the firm is operating?
\[
MP = 18L - 3L^2 = 18(5) - 3(5)^2 = 15
\]
\[
AP = \frac{9L - L^2}{L} = \frac{9(5) - 5^2}{5} = 20
\]
15/20 < 1 \Rightarrow at stage II

Suppose L = 3, at which stage the firm is operating?
\[
MP = 18L - 3L^2 = 18(3) - 3(3)^2 = 27
\]
\[
AP = \frac{9L - L^2}{L} = \frac{9(3) - 3^2}{3} = 18
\]
27/18 > 1 \Rightarrow at stage I

c. Diminishing returns starts when MP reaches its maximum; i.e., when dMP/dL = 0
\[
MP = 18L - 3L^2
\]
\[
dMP/dL = 18 - 6L = 0 \Rightarrow L = 18/6 = 3
\]

• Example:
\[
Q = 8L - 0.5L^2
\]
(Not that power 2 determines only 2 stages of production: II and III)
Law of diminishing returns starts immediately when production begins

Stage II: AP = 8 -0.5L
dAP/dL = -0.5 \Rightarrow L = 0
Stage II starts where L = 0

Stage III: MP = dQ/dL = 8 – L = 0
\Rightarrow L = 8
Stage III starts at 8

• To find AP = 0
\[
8 - 0.5L = 0 \Rightarrow L = 8/0.5 = 16
\]
Notice that AP = 0 is at twice level of L than when MP = 0
Optimal level of the Variable input:
• What level of input within Stage II is best for the firm?
• The answer depends upon:
  1. how many units of output the firm can sell,
  2. the price of the product, and
  3. the monetary costs of employing the variable input.

The Optimal quantity of the variable input is the quantity that allows the firm to maximize its profits.
• The question facing the manager is: what is the optimal number of workers?
• To make things easy, let us assume that the firm buys its inputs and sells its products in competitive markets, which means, it can hire any number of workers at the market going wage \((W)\) and sell any quantity of its product at the market going price \((P)\).

Before we continue our analysis it might be useful to go over the following definitions
• Total revenue product (TRP) refers to the market value of the firm’s output, computed by multiplying the total product by the market price \((Q \cdot P)\)
• Marginal revenue product of labor (MRP) is the change in the market value of the firm’s output resulting from one unit change in the number of workers used \((\Delta TRP/\Delta L)\).
• It can be also computed by multiplying the marginal product by the product price \((MP \cdot P)\)

Total labor cost (TLC) refers to the total cost of using the variable input, labor, computed by multiplying the wage rate (which assumed to be fixed) by the number of variable input employed \((W \cdot L)\)
• Marginal labor cost (MLC) is the change in total labor cost resulting from one unit change in the number of workers used \((\Delta TC/\Delta L)\)

Now we may present the firm profit function in the following form:
\[
\pi = TR - TC = PQ - (FC + WL)
\]
Where:
\(P\) is the Price of the good which is assumed constant,
\(Q\) is the total product,
\(FC\) is the fixed input cost,
\(W\) is the labor wage which is assumed constant, and
\(L\) is the number of workers.

Now, the question is how many workers the firm should hire to maximize its profits?
• To find out the answer, we should take the first derivative of the profit function with respect to \(L\), set it equal to zero and solve for the value of \(L\) as follows:
\[
\frac{\partial \pi}{\partial L} = P \cdot Q - W = 0 \quad \text{or} \quad P \cdot MP_L = W
\]
• Which says that, the optimal number of workers is that number at which the value of the marginal product of labor is equal to the market wage rate;
In other words, a profit-maximizing firm operating in perfectly competitive output and input markets will be using the optimal amount of an input at the point in which the marginal revenue product of labor (MRPL) equals the marginal labor cost (MLC).

\[ \text{MRP} = \text{MLC} \] or \[ P \times \text{MP} = W \]

If \( \text{MRPL} > \text{MLC} \) (or \( W \)) \( \Rightarrow \pi \uparrow \) as \( L \uparrow \)

If \( \text{MRPL} < \text{MLC} \) (or \( W \)) \( \Rightarrow \pi \downarrow \) as \( L \uparrow \)

If \( \text{MRPL} = \text{MLC} \) (or \( W \)) \( \Rightarrow \pi \) is maximum \( \Rightarrow \text{optimal input level} \)

As you may notice here again we are comparing marginal revenues and marginal cost.

Therefore, under competition in the labor market, the decision rule is to hire more workers as long as the value of the production contributed by the additional worker (\( P \times \text{MP}_L \)) exceeds the cost of hiring an additional unit of labor which is equal to the wage (\( W \)) paid to worker.

Example:
Suppose the firm short run production function has the quadratic form:
\[ Q = 100L - L^2, \]
the firm hires any number of workers at a wage of $80, and sells any quantity of its output at a price of $2.
Find how many workers should this firm hire to maximize its profits.

Solution:
The optimal number of workers is reached when \( P \times \text{MP}_L = W \), Or
when
\[ 2 \times (100 - 2L) = 80 \]
\[ 200 - 4L = 80 \]
\[ 50 - L = 20 \]
\[ \Rightarrow L = 30 \text{ workers} \]

The MRPL curve thus shows the negative relationship between the wage rate and the number of labor demanded.

At a constant wage rate, the higher the price of the output is the greater the number of workers demanded would be.

For this reason, the demand for labor (MRP) is considered a derived demand from the demand for the output.
The Optimal Mix of Variable Inputs:
• Suppose the firm production function has more than one variable input in the short run.
• In order to maximize profits, the manager has to choose the quantities of each input that will minimize cost.
• Let the production function be as follows: $Q = f(L, K, \text{Fixed input})$
Where L and K are the two variable inputs
Let $C_L$ be the cost of L and $C_K$ be the cost of K, while P is the price of the firm output.

Now by taking the first derivative of the profit function with respect to each of the variable inputs and equate it to zero we get:

$\pi = TR - TC$
$= P*Q - (C_L * L + C_K * K) - FC$
Since $Q = f(L, K)$

$\pi = P*f(L,K) - (C_L * L + C_K * K) - FC$

\[
\begin{align*}
\frac{\partial \pi}{\partial L} &= P*\frac{\partial Q}{\partial L} - C_L = 0 \\
\frac{\partial \pi}{\partial K} &= P*\frac{\partial Q}{\partial K} - C_K = 0
\end{align*}
\]

From (1) and (2)
$P*MP_L = C_L$  (3)
$P*MP_K = C_K$  (4)
Divide (3) by (4)
$MP_L = \frac{C_L}{C_K}$
$MP_K = \frac{C_K}{C_K}$

This equation can be re-written as
$\frac{MP_L}{C_L} = \frac{MP_K}{C_K}$

• Which Means to maximize profits, the marginal product generated by spending one dinar should be the same if the dinar is spent on input L or K.
• In general, optimal multiple input level is attained where the ratio of the marginal product of one input to its cost is equal to the ratio of the marginal product of the other input(s) to their cost. That is:
$\frac{MP_{X_1}}{C_{X_1}} = \frac{MP_{X_2}}{C_{X_2}} = \frac{MP_{X_3}}{C_{X_3}}$
• Other factors may outweigh this relationship such as Political/Economic risk factors.

• Example:
• Suppose you are the production manager of a company that makes computer parts in Malaysia and Algeria.
• At the current production levels and inputs utilization you found that:
  - Malaysian marginal product of labor (MP_M) = 18 Units
  - Algerian marginal product of labor (MP_A) = 6 Units
  - Wage rate in Malaysia $W_M = $ 6/hr
  - Wage rate in Algeria $W_A = $ 3/hr
• In which country should the firm hire more workers?
• Solution
Looking at the wage rates you might be tempted to hire more workers and expand production in Algeria, where wages are relatively lower.
However, by examining the MP per dollar in each country, you will find that:
\[
\frac{MP_M}{W_M} = \frac{18}{6} = 3 > \frac{MP_A}{W_A} = \frac{6}{3} = 2
\]
Which means that: an additional dollar spent on labor in Malaysia would yield 3 units, but would yield only 2 units if spent in Algeria.

• Example:
Suppose labor and capital are both variable inputs and some other inputs such as land is fixed, and suppose that 
\[
MP_L = 12 \text{ units, } MP_K = 24, w = $6 \text{ and } r = $8,
\]
\[
\frac{MP_L}{w} = 12/6 = 2 \Rightarrow \text{spending one additional dollar on labor gives two units of output}
\]
\[
\frac{MP_K}{r} = 24/8 = 3 \Rightarrow \text{spending one additional dollar on capital gives three units of output}
\]
So use more capital than labor since capital is cheaper per dollar spent than labor (capital is more productive)

• Example:
A firm has the following production function:
\[
Q = 20E - E^2 + 12T - 0.5T^2;
\]
Where E is the number of engineers and T is the number of technicians.
The average annual salary for engineers is BD 9600; and for technicians is B.D. 4800.
The firm budget for hiring engineers and technicians is BD 336000 per year.
• Calculate the optimal number of engineers and technicians.

Solution
The budget constraint:
\[
336000 = 9600E + 4800T \tag{1}
\]
The optimization condition:
\[
\frac{MP_E}{C_E} = \frac{MP_T}{C_T} \Rightarrow \frac{20-2E}{9600} = \frac{12-T}{4800} \tag{2}
\]
So, 10 – E = 12 – T or E = (T - 2) \tag{3}
By substituting (3) in (1) then;

The Long-Run Production Function
• As you know by now, the long run is a period of time long enough to allow the firm to change all its inputs. Effectively, all inputs are variable.
• As the firm increases all its inputs in the long run, it actually changes the scale of its production activity.
• The total elasticity of production, the increase in production in response to the firm proportional increase in the scale of the production process is called returns to scale.
If all inputs into the production process are doubled, three things can happen:

1. Output can be more than double, increasing returns to scale. More specialization, purchase of more sophisticated machinery.
2. Output can exactly double, constant returns to scale.
3. Output can be less than double, decreasing returns to scale. Operating on a larger scale might create certain managerial inefficiency.

Graphically, the returns to scale concept can be illustrated using the following graphs.

One way to measure returns to scale is to use a coefficient of output elasticity:

\[ E_Q = \frac{\% \Delta Q}{\% \Delta \text{ in all inputs}} \]

- If \( E_Q > 1 \), production function shows increasing returns to scale (IRTS)
- If \( E_Q = 1 \), production function shows constant returns to scale (CRTS)
- If \( E_Q < 1 \), production function shows decreasing returns to scale (DRTS)

Example:

If \( Q = 5L + 7K \); and \( L = 10 \) & \( K = 10 \)
\[ Q_1 = 5(10) + 7 (10) = 120 \text{ units} \]
Now if each input increases by 25%, then \( L = 12.5 \) & \( K = 12.5 \)
\[ Q_2 = 5 (12.5) + 7 (12.5) = 150 \text{ units} \]
\[ % \Delta Q = (150 - 120)/120 = 25\% \]
A 25% increase in \( L \) & \( K \) led to a 25% increase in \( Q \) ⇒ CRTS
\[ E_Q = \frac{\% \Delta Q}{\% \Delta \text{ in all inputs}} = \frac{25\%}{25\%} = 1 \]

Example:

\[ Q = 50X^2 + 50Y \]
\( X=1, Y=1 \) ⇒ \( Q = 50 + 50 = 100 \)
If \( X=2, Y=2 \) ⇒ \( Q = 200 + 200 = 400 \)
\[ % \Delta Q = (400 - 100)/100 = 300\% \]
⇒ \( E_Q = 300\%/100\% = 3 > 1 \Rightarrow IRTS \)
### Forms of Production Functions

- **Short run**: existence of a fixed factor to which is added a variable factor
  - One variable, one fixed factor: $Q = f(L)K$
  - Increasing marginal returns followed by decreasing marginal returns
    - Cubic function: $Q = a + bL + cL^2 - dL^3$
    - Diminishing marginal returns, but no Stage I
    - Quadratic function: $Q = a + bL - cL^2$

- **Power function**: $Q = aL^b$
  - If $b > 1$, MP increasing
  - If $b = 1$, MP constant
  - If $b < 1$, MP decreasing
  - Can be transformed into a linear equation when expressed in logarithmic terms
    $$\log Q = \log a + b\log L$$

### Cobb-Douglas Production Function:

- The Cobb-Douglas production function is a non-linear power or exponential function in the following form:
  $$Q = aL^bK^{1-b}$$
- Both capital and labor inputs must exist for $Q$ to be a positive number
- $b$ and $1-b$ are the elasticities of production with respect to labor and capital.
- $b$ and $1-b$ are constants. $b + 1 - b = 1 \Rightarrow CRTS$

- In later version, Cobb-Douglas relaxed this requirement and rewrote the equation as follows
  $$Q = aL^bK^c$$
- Can be increasing, decreasing, or constant returns to scale
  - $b + c > 1$, IRTS
  - $b + c = 1$, CRTS
  - $b + c < 1$, DRTS

- $b$ and $c$ represents the partial elasticity of production:
  - $b = \frac{MP_L}{AP_L}$
  - $c = \frac{MP_K}{AP_K}$
  - So $b$ can be found using $MP_L$ and $AP_L$. Same thing for $C$.
  - Permits us to investigate MP for any factor while holding all others constant
  - Each of the coefficients is usually less than one showing that the production takes place in stage two.
  - Each exhibits diminishing marginal returns

- Can accommodate any number of independent variables
  $$Q = aX_1^bX_2^cX_3^d \ldots X_k^n$$
  - Does not require that technology be held constant
  - For Cobb-Douglas production function
  - Sum of Exponents $= 1 \Rightarrow CRTS$
  - Sum of Exponents $> 1 \Rightarrow IRTS$
  - Sum of Exponents $< 1 \Rightarrow DRTS$
• Example:
  \[ Q = 75X^{0.25}Y^{0.75} \]
  \[ X=1, Y=1 \Rightarrow Q = 75*1*1 = 75 \]
  \[ X=2, Y=2 \Rightarrow Q = 75*1.19*1.68 = 149.94 = 150 \]
  \[ \%\Delta Q = (150 - 75)/75 = 100\% \]
  \[ \Rightarrow E_Q = 100%/100% = 1 \Rightarrow CRTS \]
  
  Or
  
  just \[ 0.25 + 0.75 = 1 \Rightarrow CRTS \]

• Example:
  \[ Q = 25L^{0.35}K^{0.75} \]
  SINCE \[ 0.35 + 0.75 = 1.1 > 1 \] \Rightarrow IRTS

• Example:
  \[ Q = 100A^{0.1}B^{0.6}C^{0.5} \]
  SINCE \[ 0.1 + 0.6 + 0.5 > 1 \] \Rightarrow IRTS

• Example:
  \[ Q = 25L^{0.35}K^{0.5} \]
  SINCE \[ 0.35 + 0.5 = 0.85 < 1 \] \Rightarrow DRTS

• Shortcomings:
  1. Cannot show MP going through all three stages of production. Cubic function is necessary
  2. Cannot show a firm or industry passing through increasing, constant, and decreasing returns to scale
  3. Specification of data to be used in empirical estimates